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Mathematics for anaesthesia

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Trigonometry

Trigonometry essentially describes mathematics around right-angled triangles and circles and, by extension, to periodic functions.[1]

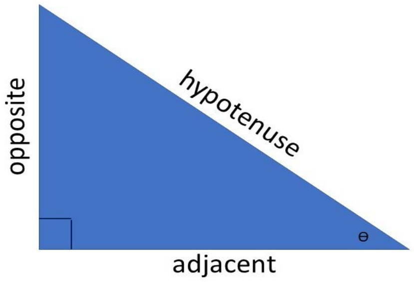

Figure 1

Figure 1 illustrates the right-angled triangle where:

- The angle we usually know is indicated by θ.
- The side opposite the right angle (the longest side) is called the hypotenuse.
- The side opposite θ is called the opposite.
- The side next to θ which is not the hypotenuse is called the adjacent.[2]

Functions for any angle Θ:

sineΘ = opposite/hypotenuse

cosineΘ = adjacent/hypotenuse

tangentΘ = opposite/adjacent

Pythagoras's theorem discusses the ratios of the sides within a right-angled triangle as follows:[2]

$(\text{Hypotenuse})^2 = (\text{adjacent})^2 + (\text{opposite})^2$

If we were to plot the right-angled triangle onto a circle, and more specifically the unit circle where radius is 1 and its centre at 0, we could demonstrate the relationships of the functions and the angle Θ.[2,3]

For example:

When Θ is 45°:

Sin(45) = 0.707

Cos(45) = 0.707

Tan(45) = 1

When Θ is 90°:

Sin(90) = 1

Cos(90)= 0

Tan(90) = undefined[4]

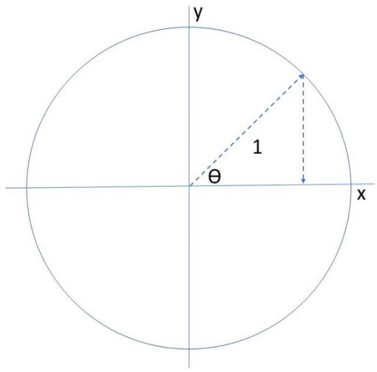

Figure 2

If a point rotates around the circumference of a circle (Figure 2) at a constant velocity and sweeps out the angle Θ, the vertical amplitude can be traced with the x axis as time, producing a periodic function or sinusoid (seen in Figure 3).[2]

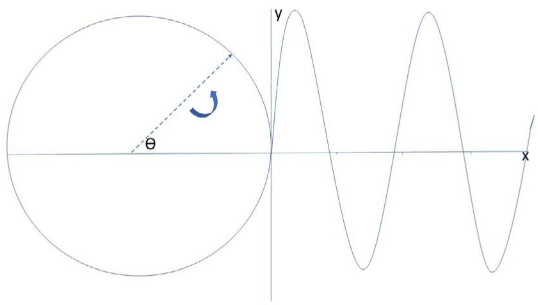

Figure 3

With regards to a circle we know that the ratio between the circumference of the circle and the diameter of the circle is described using Pi. Pi is an irrational number and is approximately = 3.14

The area of a circle = πr^2

The volume of a sphere = $4/3 \pi r^3$

The volume of a cylinder = $\pi r^2 h$. [2]

We can plot the sinusoidal waveform as sine and cosine functions (Figure 4).[4]

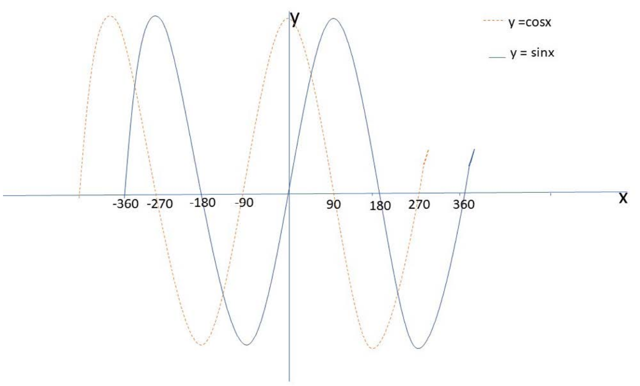

Figure 4

Note that radians may be used mathematically in place of angles, where radians = 180°, 2 radians = 360° and ½ radians = 90° and so forth.[2]

Different waveforms with different amplitudes and frequencies can be superimposed, to make more complex waveforms (ECG, or arterial line waveforms). Complex waveforms can also be broken down into composite waveforms of fundamental sinusoids (harmonics) by Fourier analysis.[5]

Inverse ratios will not be discussed in these notes.

Some algebra

Mathematical relationships are described with the use of graphs (x and y axes) and equations. Traditionally x is the independent variable, and y is the dependent variable.

Linear equations

This describes a straight line in which y will increase or decrease in proportion to x.

$y = mx + c$. If $c = 0$ then the line will pass through 0.

m is the gradient of the line or the rate of change of the dependent variable with respect to the independent variable. This is illustrated in Figure 5.[6]

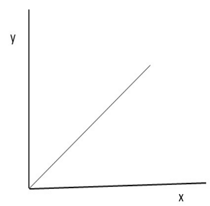

Figure 5

Quadratic equations: parabola

Describes a relationship of $y = mx^2 + c$. This is illustrated in Figure 6.

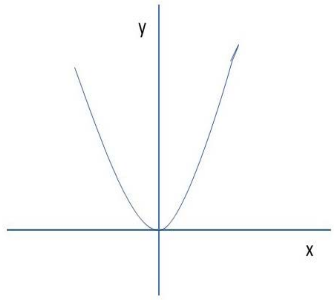

Figure 6

Exponential functions

A function is an equation where for any x-value inserted into the equation there will only be one unique resultant y-value. In terms of notation we will use *f*(x) as opposed to y in the equation where 'f(x)' denotes 'a function of x'.

Exponential growth/decay describes when something grows/decays in proportion to its current size.

This can be seen when using the equation: $f(x) = b^x$, so x (the independent variable) is no longer the base unit, but rather the exponent.[2,6]

Exponential growth is demonstrated by the curve in Figure 7: $y = b^x$

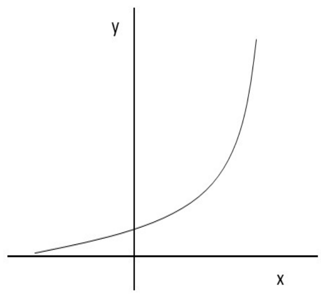

Figure 7

Exponential decay is shown in Figure 8: $y = b^{-x}$

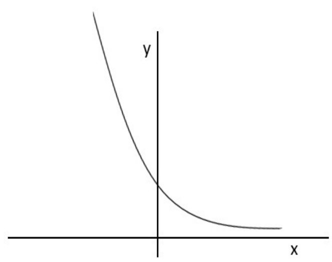

Figure 8[2,6]

Eulers constant or the letter e

Described by Euler. It is the natural number of continuous growth. If $y = 1 \times (1 + 1/n)^n$ and $n \rightarrow \infty$, then $y = e$. If on an exponential growth curve of $y = e^x$, at any point on that curve, the gradient, area under the curve and value will be e^x.[7] Euler's number is the base of natural logarithms and equates to approximately 2.718.[5]

Logarithm

Logarithm is the opposite of a power or exponent.

Exponentiation: $B^k = bxbxbxbx\ldots\ldots xb$ k times

As an example, if $b^k = c$:

If $b = 2$ and $k = 3$ then $b^k = 2^3 = c = 8$

Then likewise:

$\text{Log}_b c = k \quad \text{Log}_2 8 = 3$

The exponent is the output and the exponentiation is the input.[6]

Graphically we can see that a logarithmic function will be the inverse of an exponential function in Figure 9.

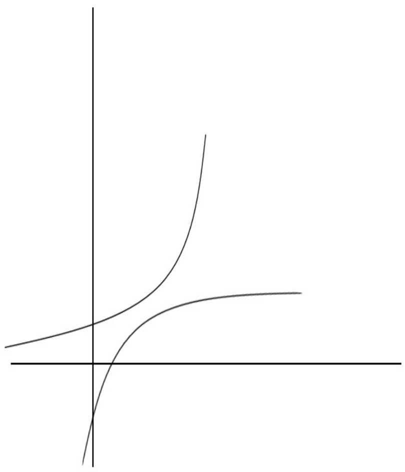

Figure 9

The use of 10 as the base is common practice and it is assumed: $\log_{10}x = \log x$

Remember the log rules:

- Product: Log [xy] = log [x] + log [y]
- Quotient: Log [x/y] = log [x] - log [y]
- Log [1] = 0
- Reciprocal Log [1/x] = –log [x].[6]

A common example of the use of a logarithmic number scale is that of hydrogen ion concentration where pH = -log (H^+).[2]

These notes serve as a basic introduction into the principles of mathematics and its application within the practice of anaesthesia, in physiology, physics and pharmacology.

Conflict of interest

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